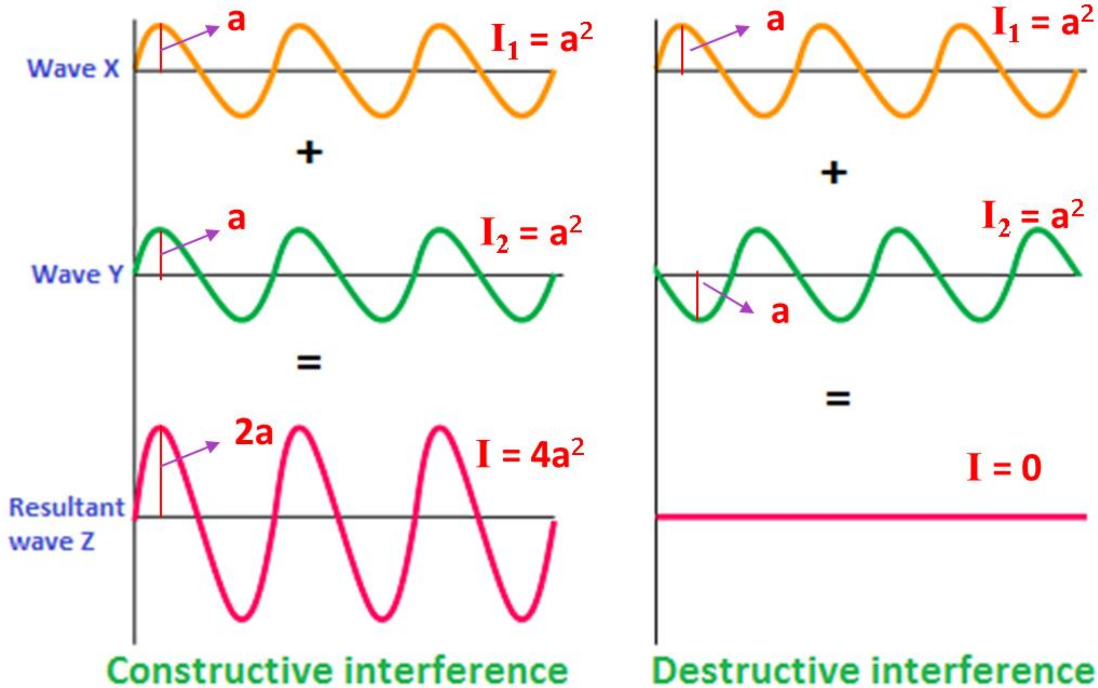
UNIT- I : INTEREFERENCE

Introduction-Principle of Superposition – Interference-Conditions for sustained interference – Coherent Sources – Interference in thin films (reflection geometry) – Newton's rings – Applications of interference.

1. Interference

The modification of amplitude or intensity of light due to the superposition of waves is known as interference.

- When the resultant intensity is sum of the intensities of individual waves, the interference is known as *constructive interference.*
- When the resultant intensity is difference of the intensities of individual waves, the interference is known as *destructive interference.*



Q1.1: Is interference pattern form in all cases or any conditions required?

According to Young's double slit experiment, in all the cases interference pattern cannot form. There are certain conditions where interference could form.

Conditions for sustained interference:

The following conditions are required to obtain stable interference pattern.

- The two sources should be coherent
- The source should be monochromatic
- The distance between two sources should be small
- The distance between source and screen should be large

- The two sources must emit continuous waves of same wavelength and same frequency
- The amplitude of interfering waves should be equal
- The sources must be narrow, i.e., they must be extremely small

1.1 Coherence

Two or more waves are in a fixed and predictable phase relationship each other said to be coherence

Coherent sources

Two sources have same frequency and have zero or constant phase difference are called coherent sources

Incoherent sources

Two sources have same frequency and not have constant phase difference are called coherent sources. e.g. sun, candle, lamp, etc.

NOTE:

Two independent sources of light can never be coherent even though they have same power, same wavelength and same amplitudes. Because, the waves emitted from two different sources will not have constant phase difference.

Hence, for practical purposes, two secondary sources of the same source act as coherent sources.

1.2 Types of coherence

Coherence are two types. 1. Temporal Coherence

2. Spatial Coherence

Temporal coherence

If the phase difference between two fields is constant during the period of time is said to have temporal coherence. This related to finite bandwidth of the source.

Spatial coherence

If the phase difference for any two fixed points in a plane normal to wave propagation does not vary with time, then wave is said to have spatial coherence.

Temporal coherence	Spatial coherence
1. If the phase difference between two fields is constant during the period is said to have temporal coherence.	1. If the phase difference for any two fixed points in a plane normal to wave propagation does not vary with time, then wave is said to have spatial coherence.
2. This is related with time	2. This is related with position
3. This is related with bandwidth of source	3. This is related with size of light source

1.3 Types of interference

The interference phenomena are classified as

(i) Division of wavefront
(ii) Division of amplitude

Division of wavefront

The wavefront divided by two parts by reflection or refraction. These two parts travel unequal distances and meet to produce interference pattern. This type of interference occurred due to division of wavefront.

e.g. Young's double slit experiment, Fresnel's Biprism experiment, etc.

Division of amplitude

The amplitude divided by two parts by reflection or refraction. These two parts travel unequal distances and meet to produce interference pattern. This type of interference occurred due to division of amplitude.

e.g. Newton's rings experiment, Michelson's interferometer experiment, etc.

Q1.2: Where did you find interference pattern in our daily life?

The best examples of interference pattern in our daily life are colors in soap bubble, colors in oil floating on water, colors in compact disc, colors in butterfly wings, colors in peacock feather, etc.

When sun light (or white light) incident on these objects, the reflected light rays superpose and produce interference pattern i.e. colors on the surface.

Q1.3: How interference pattern looks like if we replace white light with monochromatic source of light (i.e having single wavelength).

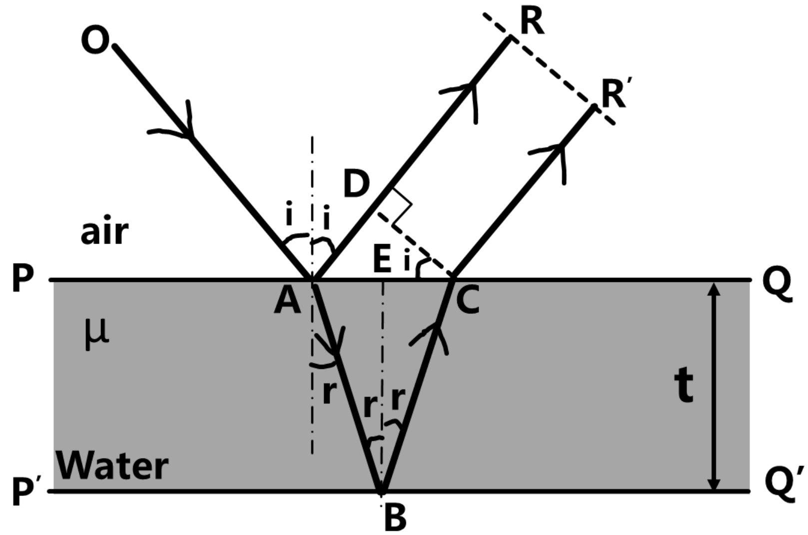
When monochromatic source of light, for example green light, incident on the soap bubble then the alternative bright and dark fringe will form.

Why and how these bright and dark fringes are forming in thin films (e.g. soap bubble, oil floating on water and compact disc) when monochromatic source of light is used? it is explained below.

2. Interference in thin films by reflection:

- Consider two parallel surfaces PQ and $P'Q'$ are separated by a distance 't'.
- The refractive index of the medium is 'μ'.
- Let a light ray OA of wavelength 'λ' incident on the surface PQ at point 'A' and reflected ray is AR.
- The refracted ray AB again at point 'B' reflected into same medium and emerge out as CR' ray as shown in figure.

- These reflected rays AR and CR' superpose each other and produce interference pattern.



- The path difference(Δ) between the rays AR and CR' $= \mu(AB + BC) - AD - \frac{\lambda}{2}$
- Here, the reflected ray undergoes phase of change of 'π' radians so that additional path difference $\frac{\lambda}{2}$ is introduced.
- From $\Delta\, AEB,\ \cos r = \frac{BE}{AB} \quad \Rightarrow \quad AB = \frac{t}{\cos r} \quad (\because BE = t)$

  Similarly, $\Delta\, BEC,\ \cos r = \frac{BE}{BC} \quad \Rightarrow \quad BC = \frac{t}{\cos r} \quad (\because BE = t)$

$$\therefore AB + BC = \frac{t}{\cos r} + \frac{t}{\cos r} = \frac{2t}{\cos r} \text{ ----------- (1)}$$

- From $\Delta\, ADC,\ \sin i = \frac{AD}{AC} \quad \Rightarrow \quad AD = AC \sin i$ ----------- (2)

  $We\ know\ that,\ \mu = \frac{\sin i}{\sin r} \quad \Rightarrow \quad \sin i = \mu \sin r$ ------------- (3)

- From $\Delta\, AEB,\quad \tan r = \frac{AE}{BE} \quad \Rightarrow \quad AE = BE \tan r = t\ \tan r$

  Similarly, $\Delta\, BEC,\quad \tan r = \frac{EC}{BE} \quad \Rightarrow \quad EC = BE \tan r = t\ \tan r$

$$\therefore AC = AE + EC = 2t \tan r$$

- $From, eqs. (2)\&(3),\ AD = 2t\ \tan r \sin i = 2t \frac{\sin r}{\cos r}\, \mu \sin r = 2\mu t \frac{\sin^2 r}{\cos r}$ ---------- (4)

- $From, eqs. (1)\&(4), Path\ difference\ (\Delta) = \mu(AB + BC) - AD - \frac{\lambda}{2}$

$$\Delta = \mu \frac{2t}{\cos r} - 2\mu t \frac{\sin^2 r}{\cos r} - \frac{\lambda}{2}$$

$$\Rightarrow \Delta = \frac{2\mu t}{\cos r}(1 - \sin^2 r) - \frac{\lambda}{2}$$

$$\Rightarrow \Delta = \frac{2\mu t}{\cos r}\cos^2 r - \frac{\lambda}{2}$$

$$\boxed{\Delta = 2\mu t \cos r - \frac{\lambda}{2}}$$

- The path difference between the rays AR and CR' $= 2\mu t \cos r - \frac{\lambda}{2}$
- We know that, condition for Maxima is $\Delta = n\lambda$, where n = 0,1,2,3,……..

$$\therefore 2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}, \quad where\ n = 0,1,2,3,\ldots..$$

Therefore, **condition for maxima** is $\boxed{\mathbf{2\mu t \cos r = (2n+1)\frac{\lambda}{2}, \quad where\ n = 0,1,2,3,\ldots..}}$

- We know that, condition for Minima is $\Delta = (2n+1)\frac{\lambda}{2}$, where n = 0,1,2, 3, …….

$$\therefore 2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t \cos r = (n+1)\lambda, \quad where,\ n = 0,1,2,3,\ldots..$$

$$2\mu t \cos r = m\lambda, \quad where,\ m = n+1 = 1,2,3,\ldots..$$

Therefore, **condition for minima** is $\boxed{\mathbf{2\mu t \cos r = m\lambda, \quad where\ m = 1,2,3,\ldots..}}$

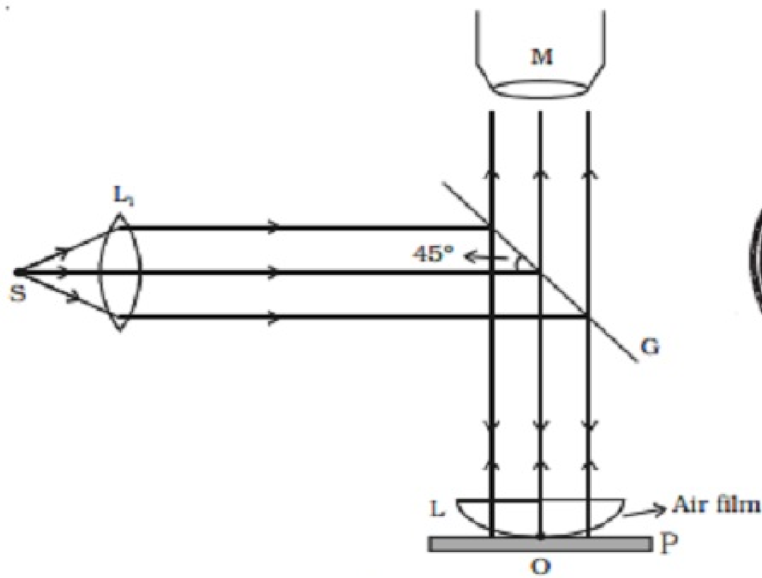
Q2.1: Why colors appear in thin films (e.g. soap bubble, oil floating on water, etc.) in presence of white light? Explain.

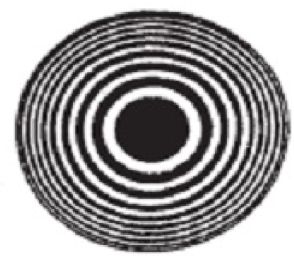
- When light incident on a thin film, the reflected rays coming from top and bottom surfaces are superpose each other and form colors in thin film.
- The condition for interference is a function of thickness of film, wavelength and angle of refraction.
- Since white light having different wavelengths, the condition for constructive interference satisfies at different points. Hence, multiple colors appear on thin film.

3. Newton's rings:

- Consider a plano-convex lens is placed on a glass plate and an air film of increasing thickness is observed between glass plate and lens as shown in figure.
- The thickness of air film at point of contact is zero.

- When monochromatic light is allowed to fall normally, the alternative bright and dark rings are formed and these are called Newton's rings.
- A plano-convex lens (L) of radius of curvature (R) is placed on a glass plate (P).
- A light from monochromatic source (S) incident on a glass plate (G) placed at an angle of 45°.
- The light rays coming from glass plate 'G' incident on the lens 'L'.
- The light rays are reflected from the curved surface of lens and glass plate 'P'.
- These reflected rays superpose each other and produce interference pattern in form of bright and dark circular rings.





- The path difference between reflected rays is $\Delta = 2\mu t \cos r + \frac{\lambda}{2}$

  Here, it is air film, $\mu = 1$ and for normal incidence, r = 0.

  Hence, path difference $\boxed{\Delta = 2t + \frac{\lambda}{2}}$

- We know that, condition for Maximum or bright is $\Delta = n\lambda, \quad where\ n = 0,1,2,3, \dots ....$

  For Bright ring, $2t + \frac{\lambda}{2} = n\lambda \quad \Rightarrow \quad \boxed{2t = (2n-1)\frac{\lambda}{2}, \quad where\ n = 1,2,3, \dots \dots ,}$

- Condition for Minima or dark is $\Delta = (2n+1)\frac{\lambda}{2}, \quad where\ n = 0,1,2,3, \dots ....$

  For Dark ring, $2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \quad \Rightarrow \quad \boxed{2t = n\lambda, \quad where\ n = 0,1,2,3, \dots \dots ,}$

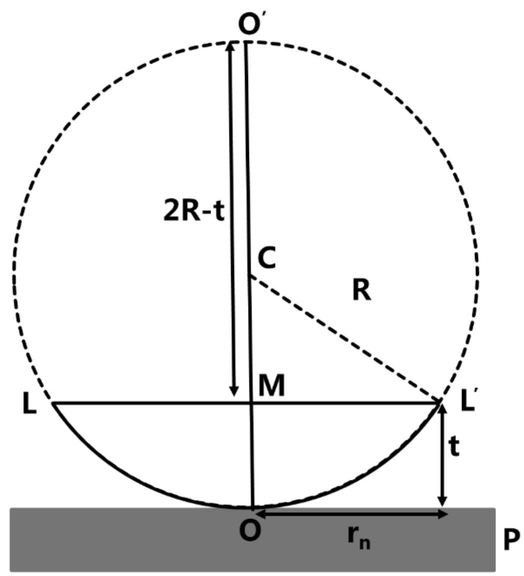
3.1 Expression for diameter of bright and dark rings:

- Let LOL' be the lens placed on glass plate 'P'. The curved surface of lens is part of sphere with radius R and center O'.
- Consider 'r_n' be the radius of Newton's ring at thickness of air film 't'.
- From figure, it is clear that $LM \times ML' = OM \times MO'$

$$\Rightarrow r_n \times r_n = t \times (2R - t)$$

$$\Rightarrow r_n^2 = 2Rt - t^2$$

$$\Rightarrow r_n^2 = 2Rt \qquad \text{-----(1)} \qquad (since\ 't'\ is\ very\ small)$$



- Condition for **Bright ring is** $2t = (2n-1)\frac{\lambda}{2}$, $where\ n = 1,2,3, \ldots \ldots$

From eq. (1), $2\frac{r_n^2}{2R} = (2n-1)\frac{\lambda}{2}$

Diameter of n[th] Newton's ring, $D_n^2 = 4\, r_n^2$

$$\frac{D_n^2}{4R} = (2n-1)\frac{\lambda}{2}$$

$$D_n^2 = \mathbf{2}\lambda R(2n-1)$$

$$\boxed{\boldsymbol{D_n} = \sqrt{\mathbf{2}\lambda R(2n-1)}\,, \quad where\ n = 1,2,3, \ldots \ldots}$$

$$\boldsymbol{D_n} \propto \sqrt{(2n-1)}$$

Thus, diameter of the bright ring is proportional to the square root of the odd natural numbers.

- Condition for **Dark ring is** $2t = n\,\lambda, \quad where\; n = 0,1,2,3, \dots\dots$

  From eq. (1), $2\,\frac{r_n^2}{2R} = n\,\lambda$

  Diameter of n^th^ Newton's ring, $D_n^2 = 4\,r_n^2$

$$\frac{D_n^2}{4R} = n\,\lambda$$

$$\boxed{D_n^2 = \mathbf{4n}\lambda R, \;, \quad where\; n = 0,1,2,3, \dots\dots}$$

$$\boldsymbol{D_n} \propto \sqrt{n}$$

Thus, diameter of the bright ring is proportional to the square root of the natural numbers.

3.2 Application of Newton's rings experiment

(i) Determination of wavelength of given light source

- Let 'R' be the radius of curvature of the plano-convex lens and 'λ' is the wavelength of light used.
- We know that, the diameter of n^{th} dark ring is $D_n^2 = \mathbf{4n}\lambda R$ -------- (1)
- If $D_{(n+P)}$ is the diameter of $(n+P)^{th}$ dark ring, then $D_{n+P}^2 = \mathbf{4(n+P)}\lambda R$ -------- (2)

  From eqs. (1) and (2), $D_{n+P}^2 - D_n^2 = \mathbf{4(n+P)}\lambda R - \mathbf{4n}\lambda R$

$$D_{n+P}^2 - D_n^2 = \mathbf{4P}\lambda R$$

Wavelength of given source of light, $\boxed{\lambda = \frac{D_{n+P}^2 - D_n^2}{\mathbf{4P}R}}$

(ii) Determination of refractive index of a liquid

- Let 'R' be the radius of curvature of the plano-convex lens and 'λ' is the wavelength of light used.
- We know that, the diameter of n^{th} dark ring is $D_n^2 = \mathbf{4n}\lambda R$ -------- (1)
- If $D_{(n+P)}$ is the diameter of $(n+P)^{th}$ dark ring, then $D_{n+P}^2 = \mathbf{4(n+P)}\lambda R$ -------- (2)

From eqs. (1) and (2), $D_{n+P}^2 - D_n^2 = \mathbf{4(n+P)}\lambda R - \mathbf{4n}\lambda R$

$$D_{n+P}^2 - D_n^2 = \mathbf{4P}\lambda R \quad \text{-------- (3)}$$

- Consider a liquid of refractive index 'μ' placed between the plano-convex lens and plane glass plate. The diameters of n^{th} and $(n+P)^{th}$ dark rings are D_n' *and* D_{n+P}' respectively.

Therefore, $D'^2_{n+P} - D'^2_n = \frac{4P\lambda R}{\mu}$ -------- (4)

From eqs. (3) and (4), $$\mu = \frac{D^2_{n+P} - D^2_n}{D'^2_{n+P} - D'^2_n}$$

Q 3.1: Why central fringe in Newton's rings experiment is always dark?

At central fringe position, thickness of air film is zero. So that path difference is equal to $\frac{\lambda}{2}$.

As t = 0, path difference $\Delta = 2t + \frac{\lambda}{2} \Rightarrow \Delta = \frac{\lambda}{2}$

This is the condition for formation of destructive interference. Hence, the central fringe is always dark.

Q 3.2: Why the fringes are circular rings in Newton's rings experiment?

- In Newton's rings experiment, a plano-convex lens placed on glass plate.
- In this arrangement, an air film formed between lens and glass plate. The thickness of air film is gradually increasing.
- Each point on the lens having same thickness of air film produce bright or dark fringe.
- The locus of all the points of same thickness produces a bright or dark circles. Likewise, alternative bright and dark rings are formed in Newton's rings experiment.

Q 3.3: What happens to the Newton's rings when monochromatic source replaced with white light in Newton's rings experiment?

In Newton's rings experiment, diameter of ring depends upon wavelength of light.

Each colored light will have different wavelength. So that, each colored ring having different diameter.

The overlapping of these rings displays colored rings when white light is used in the experiment.

Q 3.4: If an air film replaced with a liquid of refractive index 'μ', what happens to diameter of the rings?

Condition for formation of dark ring, $D_n^2 = \mathbf{4n}\lambda R,\ where\ n = 0,1,2,3,\ldots\ldots$ --------------- (1)

This condition is valid for air film.

Let us assume, when a liquid of refractive index 'μ' *is placed, the diameter changes to* D'_n then

$$D'^2_n = \frac{\mathbf{4n}\lambda R}{\boldsymbol{\mu}} \quad \text{--------------- (2)}$$

From (1) and (2), $D'^2_n = \frac{D_n^2}{\mu} \Rightarrow \boxed{D'_n = \frac{\mathbf{D_n}}{\sqrt{\boldsymbol{\mu}}}}$

Therefore, the diameter of ring decreases when liquid is placed.

4. Applications of Interference

Interference phenomenon used to

- Find the thickness of transparent materials
- Determine the refractive index of transparent solids, liquids and gases
- Determine wavelength of light
- Find the reflecting power of the lens and prism surfaces.

Numerical

1. The ratio of intensity of maxima and minima of interference fringes is 25:9. Determine the ratio between the amplitude and intensities of the two interference beams
2. Two coherent sources of intensity 16 W/m^2 and 25 W/m^2 interfere to form fringes. Find the ratio of maximum intensity to minimum intensity.
3. A parallel beam of light of wavelength 5890 Å is incident on a glass plate of refractive index 1.5 such that the angle of refraction into the plate is 60°. Calculate the smallest thickness of the plate which will make it appear dark by reflection.
4. Calculate thickness of a soap film (μ = 1.463) that will result in constructive interference in the refracted light, if the film is illuminated normally with light whose wavelength in free space is 6000 Å.
5. Newton's rings are observed in the reflected light of wavelength 5900 Å. The diameter of 10th dark ring is 0.5 cm. Find the radius of curvature of lens used.
6. In Newton's rings experiment, if diameters of 4th and 6th dark rings are found to be 3 mm and 3.6 mm, calculate the wavelength of light used. The radius of curvature of the convex lens surface of the lens is 0.9 m.
7. In Newton's rings experiment, if diameters of 4th and 12th dark rings are found to be 0.4 cm and 0.7 cm respectively. Find the diameter of the 20th dark ring.
8. Calculate thickness of the air film at 10th dark ring in Newton's rings system viewed normally by a reflected light of wavelength 500 nm. The diameter of 10th dark ring is 2 mm.